NATIONAL AIR INTELLIGENCE CENTER

THE OPTIMAL CENTRAL GUIDANCE OF MEDIUM RANGE
AIR-TO-AIR MISSILE

by

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THE OPTIMAL CENTRAL GUIDANCE OF MEDIUM RANGE AIR-TO-AIR MISSILE

by LUI HUI ZEN, WU JIAN ZHONG, DONG GUANG JIN

Symbols

C_D, C_1 ------resistance and lifting force coefficient

C_{DO} --------Zero lifting force coefficient

D ----------resistance

g ----------weight gravity

h, h_i -------height of missile and target

K ----------induced resistance coefficient

K_1, K_2 ------characteristic indicator coefficient

L ----------lifting force

Ma ---------Mach value

m ----------mass

r ----------radial distance between missile and target

t ----------time

S ----------reference area

t_t ---------intercepting time

t_{go} --------residual time

T ----------thrust

v ----------velocity

x, x_t -------horizontal coordinates of missile and target

α, α_0 -------attack angle and zero lifting force angle

γ ----------missile aiming angle

ρ ----------air density

φ --------condition indication function

Ω --------terminating condition

$(.)$ --------time derivative

i, f -------initial and final values

max, min---maximum and minimum

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1. Introduction

In recent years, the development of high speed micro-computers makes it possible to apply optimal control theory to the missile guidance system. Although, numerous research papers have been published on this subject, hardly any of them deal with the concept of central guidance control. Here, an optimal central guidance principle that satisfies the needs of missiles with various speeds and guiding time is proposed.

The major purpose of central guidance control is to guide the missile and enable it to maintain an optimal geometric position in relation to its target as soon as the target is locked on to by the missile head guidance. In order to reach the necessary maximum gravity force, the missile must have a certain minimum velocity which is contingent upon target's maximum gravity force, height and distance between them. For distant and low altitude targets, the major factor is missile' speed. It's better to use a central guidance that can maximize the residual speed of the missile. For closer targets, the timing is very important, since the missile has to destroy the target before it launches a counter attack. Therefore, central guidance control that can minimize the intercepting time has to be applied. By solving the

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boundary value of nonlinear equations, these guidance rules can be obtained.

Here, we will introduce the mathematical model of medium range air to air missiles, and discuss guidance principles of self searching. At the end, we will introduce the central guidance principle.

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* Numbers in margins indicate foreign pagination. Commas in numbers indicate decimals.

2. Mathematical Model

Fig. 1 is a diagram on force balance and symbols used. For simplification, missile movement is assumed to be point movement and is restricted in perpendicular plane, the movement equation is

$$\dot{v} = (T\cos\alpha - D)/m - g\sin\gamma$$

(2)
$$\dot{\gamma} = (L + T\sin\alpha)/mv - g/v\cos\gamma$$

(3)
$$\dot{x} = v\cos\gamma$$

(4)
$$\dot{h} = v\sin\gamma$$

In which

(5)
$$L = \frac{1}{2}\rho v^2 S C_L, \quad C_L = C_{L\alpha}(\alpha - \alpha_2)$$

(6)
$$D = \frac{1}{2}\rho v^2 S C_D, \quad C_D = C_{D0} + KC_L^2$$

Aerodynamic force derivative coeffiicients C_1, C_{D0} and K are functions of Ma, while Ma is a function of v and h. Air density is a function of h.
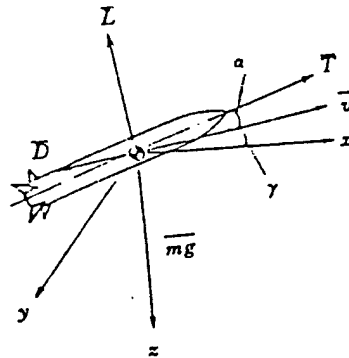
Fig.1 The force balance and symbols of the missile.

(7) $\rho=\rho(h), \quad Ma=Ma(v,h)$
        $C_{L\alpha}=C_{L\alpha}(Ma), c_{D0}=C_{D0}(Ma),$

(8) $K=K(Ma)$

The mass and thrust of missile are given as a function of time as they are shown in fig. 2.
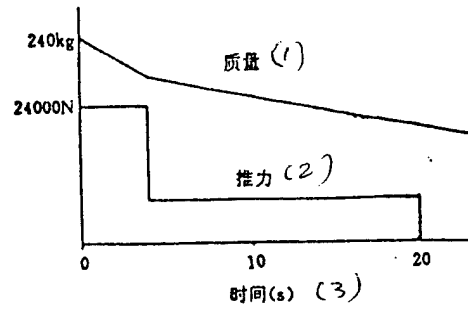
(9) $m=m(t), \quad T=T(t)$

Fig. 2 The characteristic curve of missile mass and thrust as a   function of time.

Key: (1) mass; (2) thrust; (3) times(s).

3. Guidance for Searching

Before we discuss guidance, we'd like to discuss the principle of searching in the final stages. Despite the tremendous research that has been done on the application of the guidance principle of optimal linear searching, it is still complicated to apply it to an real flight case. Recently, the application of APNG has come full circle. PNG is an outcome of the analysis of nonmobile targets, therefore, APNG can be treated as a special case of an optimally controlled mobile target. The transectional gravity a_c of this missile is

(10)
$$a_c = N_e v_c \dot{\sigma}$$
(PNG)

(11)
$$a_c = N_e (v_c \dot{\sigma} + \frac{1}{2} a_t)$$
(APNG)

N_e is effective guidance ratio, v_c is the approach speed, a_c is the transectional gravity of the target, ς is the visual spinning speed.

(12)
$$v_c = -\dot{r}$$
$$\dot{\sigma} = (1/r^2)[(h_t - h_m)(\dot{x}_t - \dot{x}_m)$$

(13)
$$-(h_t - h_m)(x_t - x_m)]$$

(14)
$$r = [(x_t - x_m)^2 + (h_t - h_m)^2]^{1/2}$$

In equation (11), APNG requires estimating the transectional gravity of the target(usually with a Karman wave filter). Acquiring this estimated value will simplify the application of a PNG missile in APNG. Fig. 3 is a simulation diagram of PNG missile in APNG.The x coordinate is the residual time of the target when it starts maximum gravity turning. The initial relative distance is 5 km, the initial speed of missile and plane is 600 m/s and 300 m/s respectively. They are both at a height of 300 m. The PNG missile has a maximum deviation from the target 1-2 s

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before the plane starts sequential maximum gravity turning(SMGT), while APNG tends to have similar action, but the deviation becomes much smaller. This is because equation(11) contains an uncertainty item of the target gravity. The self searching period of APNG will be discussed in combination with the optimal central guidance principle.
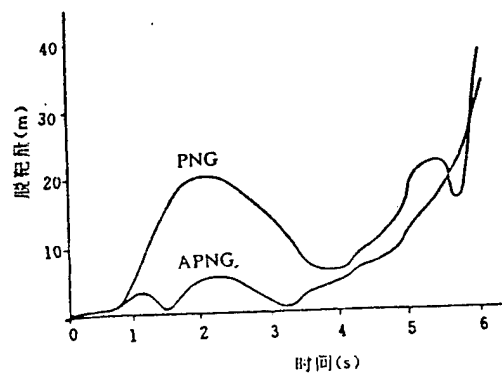

Fig.3 Missile deviation simulation based on PNG and APNG guidance principle (v_0=600 m/s, height is 5 km, locked distance is 5 km).

4. Central Guidance

Target models such as conventional plane(plane A) and advanced plane(plane B) have different transectional maximum gravity values. Fig.4 indicates the available transectional maximum gravity for the two planes. Plane A has 7g at sea level, plane B has more than 9g at heights below 3 km. In consideration of the endurance of the pilots, the maximum gravity is limited to 9g.

When the guidance head is locked on the target, the residual speed of the missile has to meet certain requirements in order to reduce the deviation to below 10 m. For the transectional maximum gravity of the target shown in fig.4, the calculation results at various geometric positions between the missile and target are shown in fig. 5 and 6. x coordinate is the locked distance when the missile just starts searching assuming the missile and target are at the same height. Usually, a greater residual speed is needed when a missile is dealing with a target that has a greater initial speed or a lower height.
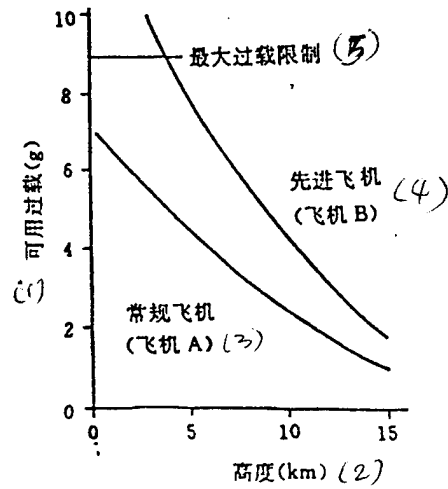
Fig.4 The available transectional maximum gravity of a plane

Key: (1) available maximum gravity; (2)height(km); (3) conventional plane(plane A);(4) advanced plane(plane B); (5) the limit for maximum gravity.

Previous studies have indicated that it is more advantageous to use downward mobility of gravity than horizontal or upward mobility. But a plane may not have enough time to pull up when it is at a low height, therefore, it is safe to assume that the plane should use maximum gravity turning at heights below 10 km; At heights that are above 10 km, it should use the superior downward maximum gravity turning(half rotation backward turning), this will make it easier for it to pull up.

Fig. 7 is an another description of fig.5 and 6. It indicates the required minimum speed of the missile in relation to its height when the targeted planes both have initial distance of 5 km. At a height of 5 Km, the head guidance requires speeds of 590 m/s and 790 m/s in order to lock on plane A and B.

Such a minimum speed will produce enough pressure to provide the necessary maximum gravity. Fig. 8 indicates the necessary maximum gravity of the missiles in relation to the heights of the two planes A and B at an attack angle α =10.

Let's suppose the intercepting point is determined by the ground support system; the missile is guided by its maximum final speed or its minimum intercepting time. Mathematically speaking, the optimal guidance principle is to reach characteristic parameter at an attack angle α (t) according to kinetic equation(1-9).

(15)
$$\varphi = (-K_1 t + K_2 v)_{tf}$$

In which the final time t_f is determined by the following terminal condition

(16)
$$\Omega \equiv 1/2[(x-x_t)^2 + (h-h_t)^2 - r_t^2]$$
$$= 0$$

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In which r_f is the locked distance of the head guidance. By changing the values of K_1 and K_2 in equation(15), an optimal control expression is obtained.

When the final speed reaches a maximum(i.e. v_f is maximum)

(17) $K_1 = 0, K_2 = 1$

When the final time is a minimum(i.e. t_f is minimum)

(18) $K_1 = 1 \quad K_2 = 0$

This equation can be solved by using the optimal descent method.

In order to maximize the final speed, we can find at least 2 optimal controls of regional maximum values. In the first category, the missile starts sudden climbing at the beginning and then immediately goes down. This enables missile acquired maximum terminal speed when it faces distant objects, this is type I. In the second category type II, the missile climbs slowly and then goes down. This enables the missile to have a maximum final speed that is still smaller than the one of type I, but it has a shorter guidance time.
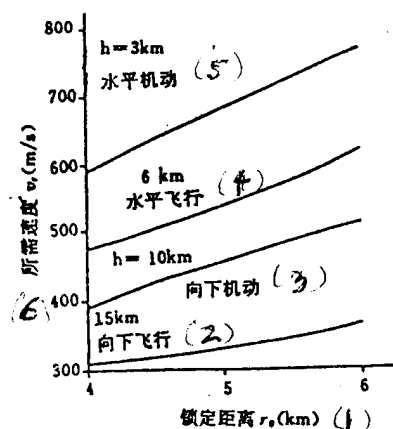

Fig. 5 The relationship between the required missile speed and the locked distance and the height(aiming plane A).

Key: (1) the locked distance r_o(km); (2) downward flying; (3) downward mobility; (4) horizontal flying; (5) horizontal mobility; (6) the required speed v_f(m/s).
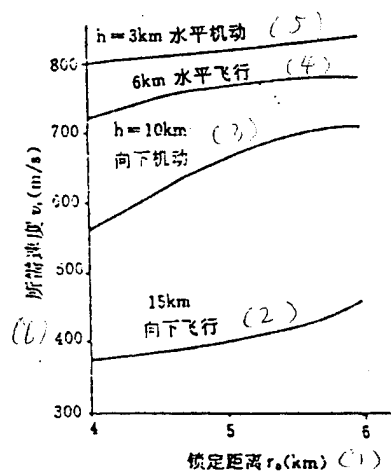
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Fig. 6 The relationship between the required missile speed and the locked distance and the height(aiming plane B).
(1) the locked distance r_o(km); (2) downward flying; (3) downward mobility; (4) horizontal flying; (5) horizontal mobility; (6) the required speed v_f(m/s).
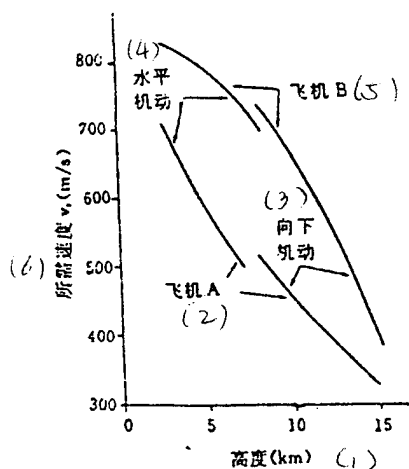

Fig.7 The relationship between the required minimum speed and the height(locked distance=5 km)
(1) height(km); (2) plane A; (3) downward mobility; (4) horizontal mobility; (5) plane B; (6) the required speed v_f(m/s).
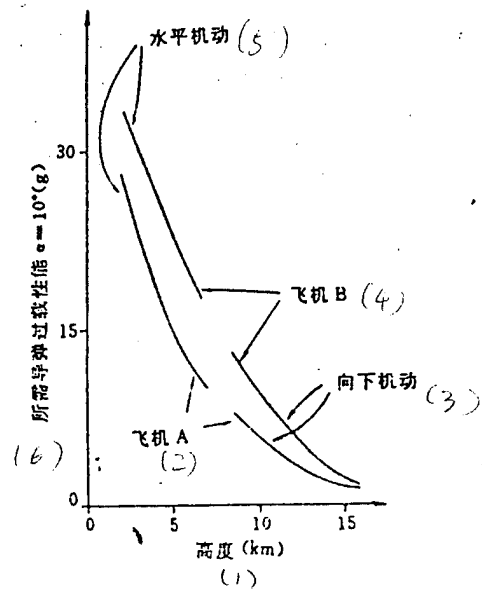
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Fig. 8 The relationship between the required maximum gravity
and the height of the missile(the locked distance=5 km).
(1) height(km); (2) plane A; (3) downward mobility; (4) plane
B(4); (5) horizontal mobility; (6) the required maximum gravity
of the missile =10 (g).

The optimal guidance principle in which the final time t_f is minimum is categorized as type III. Although, all these optimal controls have their own advantages, calculating missile controls is a very time consuming process even if a high speed modern computer is used, therefore, they can not be used for self searching guidance. The guidance principle for type I and III can only be applied to central guidance: the guidance from the launching site to the near intercepting points where the guidance head is still capable of locking on the target. After that, APNG can be used for the guidance of self searching. In comparison to the optimal control principles described above, type IV guidance principle uses an APNG missile introduced into the intercepting process.

In summary, the four guidance principles are:

Type I: maximize the missile's final speed so that it can climb rapidly and then go down;

Type II: maximize the missile's final speed so that it can climb slowly and then go down;

Type III: minimize the missile's final time;

Type IV: APNG(A represents its advanced guidance principle).

Fig. 9 indicates the theoretical intercepting track of these four guidance principles. A is the missile's initial launching site, B is the targeting site that is within the range of the locked distance of the missile. In these studies, the locked distance is assumed to be 5 km. Generally speaking, the height of the missile track using type I control is higher than for the other types.

5. Simulation results and discussion

A comparison on the speed and guidance time of the missile that has reached the end of the locked distance has been done. Fig. 10 is the typical track and control process of a missile using type I-IV guidance principles.

Fig. 11-14 indicates the relationship between v_f, t_f and horizontal distance using type I-IV guidance principles.

In this case, the missile is launched from 6 km height. The maximum horizontal distance ensures a maximum v_f using the type I guidance principle, but the final time t_f gets longer. v_f values at a bigger distance for the guidance principle type II are smaller than the ones of type I, but bigger than the ones of type III and IV, while t_f values increase only slightly. Type III has a minimum t_f values. Type IV has similar t_f, but its v_f is smaller than type II.
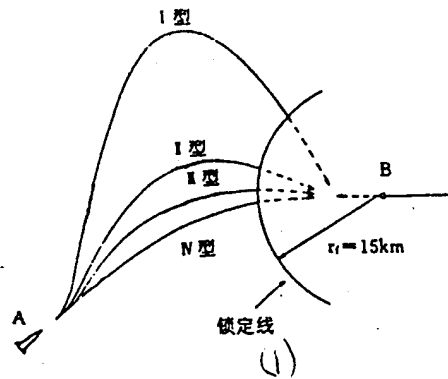
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Fig. 9 The theoretical intercepting tracks using type I-IV guidance principles.
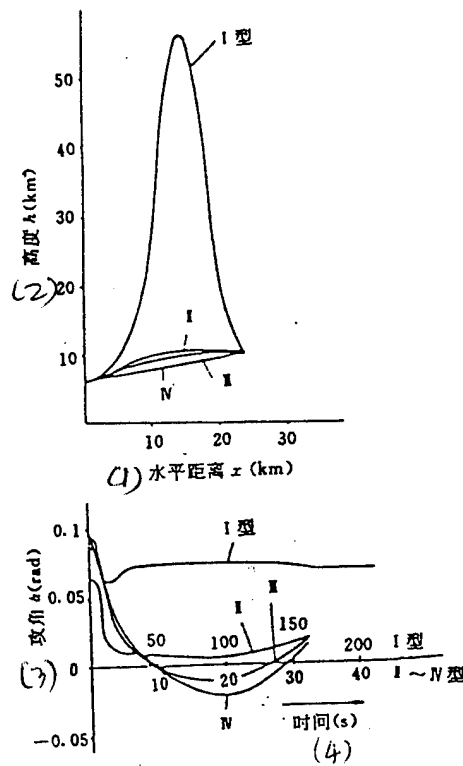
Key: (1) the locked line.

Fig. 10 The missile tracks and control process using type I-IV guidance principles(the launching height is 6 km, x_i=24 km).

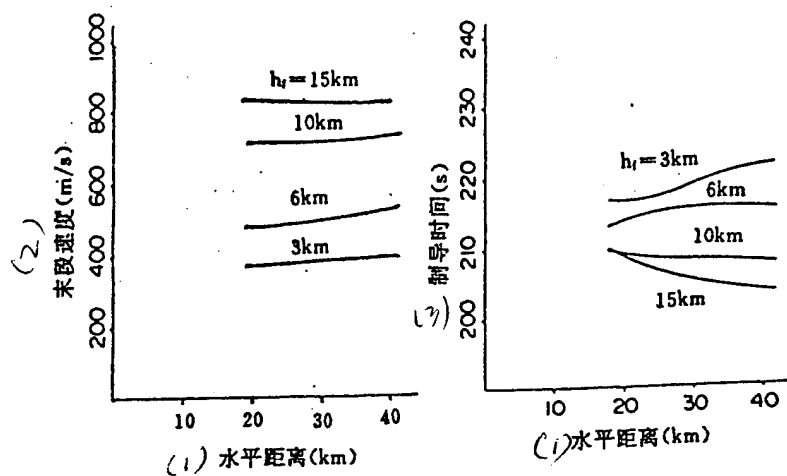Key: (1) horizontal distance x(km); (2) height h(km); (3) attack angle (rad); (4) time(s).

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Fig. 11 The relationship between the final speed v_f, the guidance time t_f and the horizontal distance x_i, the final height h_t (using type I guidance principle, the launching height is 6 km).

Key: (1) horizontal distance(km); (2) final speed(m/s); (3) guidance time(s).

Fig. 12 The relationship between the final speed v_f, the guidance time t_f and the horizontal distance x_i, the final height h_t (using type II guidance principle, the launching height is 6 km).

Key: (1) horizontal distance(km); (2) final speed(m/s); (3) guidance time(s).

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(2) 末段速度(m/s) 1000 800 600 400 200

h_f=15km
10km
6km
3km

(3) 制导时间(s) 100 80 60 40 20

6km
h_f=3km
10km
15km

(1) 水平距离(km) 10 20 30 40

(1) 水平距离(km) 10 20 30 40

Fig. 13 The relationship between the final speed v_f, the guidance time t_f and the horizontal distance x_i, the final height h_t (using type III guidance principle, the launching height is 6 km).

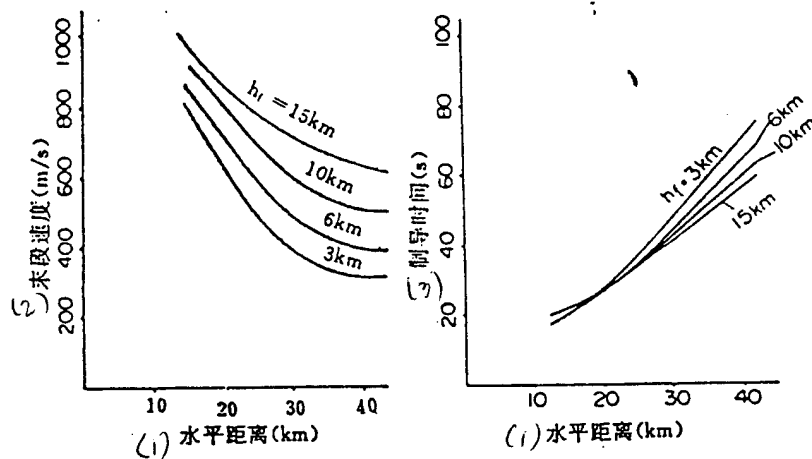Key: (1) horizontal distance(km); (2) final speed(m/s); (3) guidance time(s).

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(2) 末段速度(m/s) 1000 800 600 400 200

h_f=15km
10km
6km
3km

(3) 制导时间(s) 100 80 60 40 20

6km
10km
h_f=3km
15km

(1) 水平距离(km) 10 20 30 40

(1) 水平距离(km) 10 20 30 40

Fig. 14 The relationship between the final speed v_f, the guidance time t_f and the horizontal distance x_i, the final height h_t (using type IV guidance principle, the launching height is 6 km).

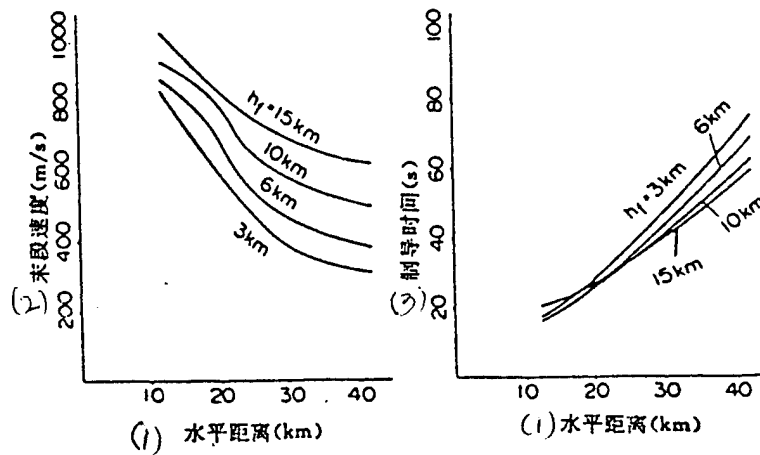Key: (1) horizontal distance(km); (2) final speed(m/s); (3) guidance time(s).
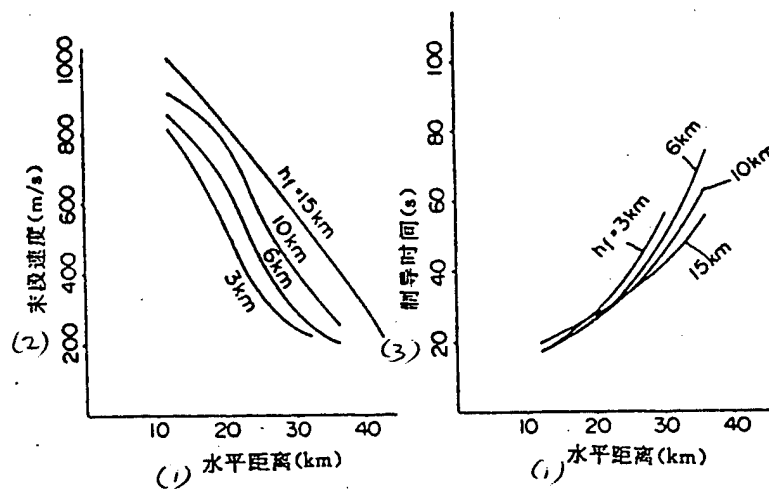
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Judging from the above results, the type I guidance principle should be used so that there will be enough time for the missile to intercept when the target is detected from far away. This ensures the missile to have a maximum v_f and a maximum gravity during the stage of self searching. However, when the target is detected from a shorter distance and there is only a very short time for it to intercept, type III guidance principle should be applied. When a missile is given enough time to intercept a target, type II guidance principle should be applied which enables it to have a bigger v_f than the ones of type III and IV. For closer targets, v_f value of type II is bigger than the one of type I. Certainly in this case, type II not type I should be used.

We recommend the use type I-III guidance principles alternatively in the central guidance stage, and the use of APNG in the searching stage. In order to illustrate the characteristics of such guidance principle, we combined fig.11-13 and fig. 7 to calculate the launching range of the missile. The type of plane and the locked height h_f are given, the required speed v_r needed for the missile to lock on the target can be found in fig. 7. Applying this known value v_r in fig. 11-14, the horizontal distance x_i can be obtained. By the same token, for a known value of x_i, the guidance time t_f can be obtained. If the plane has an approach speed of 300 m/s, the approximate value of the horizontal distance from the plane position to the borderline is $x_i + 0.r_f + v_f$(locked distance). Fig. 15 and 16 indicate the blind region that is below or at the left of 6km height launching border line where the missile can not destroy the target when dealing with type A and B plane using type IV guidance principle. This is due to the low speed of the missile, even if the guidance head can lock on the target, the restriction on firing plane B is much greater than the one on the plane A. Although, the launching area for the recommended guidance principle is considerably greater than the one using type IV. In fig. 15 and 16, when the target horizontal distance

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is smaller than 20 km, we choose type III guidance principle;
when the target horizontal distance is greater than 85 km, we
choose type I guidance principle. When the target horizontal
distance is between 20-85 km, we choose type II guidance
principle. Due to the minor difference between the type II and
type III guidance principle, we can omit type II and use the type
III guidance principle instead. If a plane locked on a target
that is outside the intercepting region, we must wait till the
target enters into areas which are above or on the left of the
missile launching borderline.

6. Conclusion

It is suggested that we should alternatively use these
guidance principles that can maximize the final speed or minimize
the final time during the central guidance stage of a missile;
while during the self searching stage, we should use the
increasing ratio guidance principle.

By virtue of optimally solving the final speed in two
maximal cases, only one of these three guidance principles should
be used in actual cases(two of these have maximal final speed,
the third one has minimum guidance time).

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The restriction on launching border when dealing with an
advanced plane is much stricter than the one encounters when
dealing with a conventional plane. In comparison to type IV
guidance principle, the suggested guidance principle considerably
expands the launching area.

飞机A (3)

高度(km)

IV型(D)   II型(D)

导弹发射点 (2)

IV型(H)   I   II型(D)

I型(D)   I型(H)
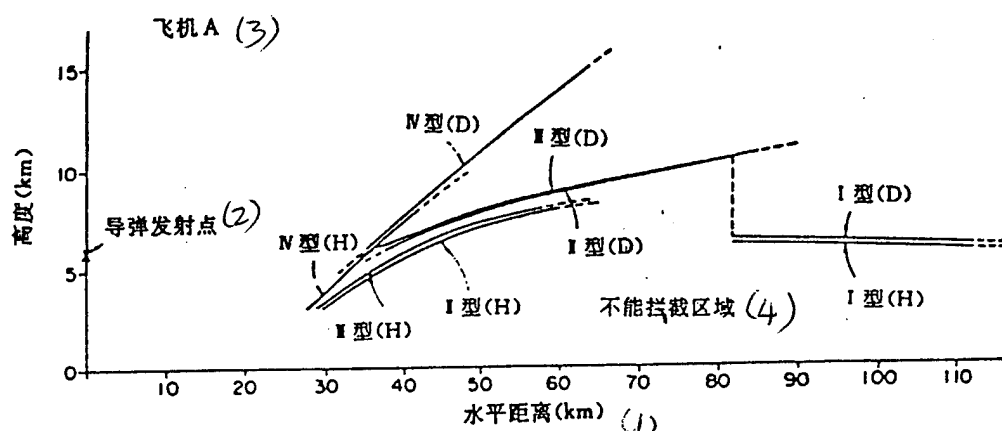
I型(H)   不能拦截区域 (4)   I型(H)

II型(H)

水平距离(km) (1)

Fig. 15 The launching border of a missile at 6 km height using type I-IV guidance principle(when dealing with plane A). H: horizontal mobility; D: downward mobility

Key: (1) horizontal distance(km); (2) missile launching site; (3) plane A; (4) uninterceptable region.

飞机B (3)

II型(D)

高度(km)

IV型(D)

导弹发射点 (2)   I型(D)

IV型(H)   II型(H)   I型(H)

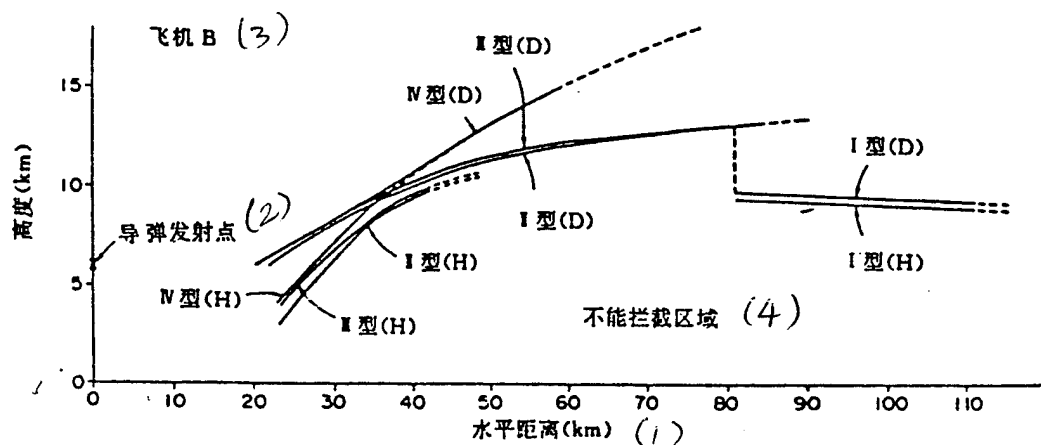II型(H)   不能拦截区域 (4)   I型(H)

水平距离(km) (1)

Fig. 16 The launching border of a missile at 6 km height using type I-IV guidance principle(when dealing with plane B).

Key: (1) horizontal distance(km); (2) missile launching site; (3) plane B; (4) uninterceptable region.

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